

STRUCTURE OF THE CIRCUMNUCLEAR REGION OF SEYFERT 2 GALAXIES REVEALED BY RXTE HARD X-RAY OBSERVATIONS OF NGC 4945

G. Madejski^{1,2}, P. Życki³, C. Done⁴, A. Valinia^{1,2}, P. Blanco⁵, R. Rothschild⁵, B. Turek^{1,6}

¹ Laboratory for High Energy Astrophysics, NASA/GSFC, Greenbelt, MD 20771, USA

² Dept. of Astronomy, University of Maryland, College Park

³ Nicolaus Copernicus Astronomical Center, Bartycka 18, 00-716 Warsaw, Poland

⁴ University of Durham, Physics Dept., Durham, DH1 3LE, UK

⁵ Center for Astrophysics and Space Sciences, Univ. of California / San Diego, LaJolla, CA

⁶ Dept. of Physics, Stanford University, Palo Alto, CA

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ABSTRACT

NGC 4945 is one of the brightest Seyfert galaxies on the sky at 100 keV, but is completely absorbed below 10 keV, implying an optical depth of the absorber to electron scattering of a few; its absorption column is probably the largest which still allows a direct view of the nucleus at hard X-ray energies. Our observations of it with the Rossi X-ray Timing Explorer (RXTE) satellite confirm the large absorption, which for a simple phenomenological fit using an absorber with Solar abundances implies a column of $4.5^{+0.4}_{-0.4} \times 10^{24} \text{ cm}^{-2}$. Using a more realistic scenario (requiring Monte Carlo modeling of the scattering), we infer the optical depth to Thomson scattering of ~ 2.4 . If such a scattering medium were to subtend a large solid angle from the nucleus, it should smear out any intrinsic hard X-ray variability on time scales shorter than the light travel time through it. The rapid (with a time scale of \sim a day) hard X-ray variability of NGC 4945 we observed with the RXTE implies that the bulk of the extreme absorption in this object does *not* originate in a parsec-size, geometrically thick molecular torus. Limits on the amount of scattered flux require that the optically thick material on parsec scales must be rather geometrically thin, subtending a half-angle $< 10^\circ$. This is only marginally consistent with the recent determinations of the obscuring column in hard X-rays, where only a quarter of Seyfert 2s have columns which are optically thick, and presents a problem in accounting for the Cosmic X-ray Background primarily with AGN possessing the geometry as that inferred by us. The small solid angle of the obscuring material, together with the black hole mass (of $\sim 1.4 \times 10^6 M_\odot$) from megamaser measurements, allows a robust determination of the source luminosity, which in turn implies that the source radiates at $\sim 10\%$ of the Eddington limit.

Subject headings: galaxies: individual (NGC 4945) – galaxies: Seyfert X-rays: galaxies

1. Introduction

Our current best picture of nuclei of Seyfert galaxies includes the central source (i.e. black hole, accretion disk and broad line region) embedded within an optically thick molecular torus (cf. Antonucci & Miller 1985). The object is classified as a Seyfert 1 for viewing directions which lie within the opening angle of the torus, so that there is a direct view of the nucleus, and as a Seyfert 2 for directions intersecting the obscuring material. The torus absorbs the optical, UV and soft X-ray nuclear light, so the nucleus in Seyfert 2s can only be seen at these energies through scattered radiation. One of the central questions still under debate in the context of the Unification Models of the two types of Seyferts is: how optically thick is this putative torus?

NGC 4945 is a nearby (3.7 Mpc; Mauersberger et al. 1996) edge-on galaxy. It has strong starburst activity, producing intense IR emission concentrated in a compact nuclear region (Rice et al. 1988; Brock et al. 1988), and a “superwind” outflowing along the minor axis of the galaxy (Heckman, Armus, & Miley 1990). It also has an active nucleus, first seen unambiguously in Ginga X-ray observations (Iwasawa et al. 1993), confirming the Seyfert 2-type classification. These data showed a heavily obscured, strong hard X-ray source above 10 keV, confirmed by the CGRO OSSE observations (Done, Madejski, & Smith 1996) which in turn revealed that NGC 4945 is one of the brightest extragalactic sources in the sky at 100 keV! The absorbing column, a few $\times 10^{24}$ cm⁻², is among the largest which still allows a direct view of the nucleus at hard X-ray energies. If such a scattering medium were to subtend a large solid angle from the nucleus, it would smear out any intrinsic hard X-ray variability on timescales shorter than the light travel time through it.

The presence of the Seyfert nucleus is further supported by the fact that the object is a megamaser source (detected in the H₂O bands) implying an edge-on geometry, but one of the key features which makes its study so important is that it is one of only four AGN where the black hole mass can be constrained (at $\sim 1.4 \times 10^6 M_{\odot}$) from detailed mapping of the megamaser spots (Greenhill, Moran, & Herrnstein 1997). As such, it is one of a few unique sources where the luminosity in Eddington units can be reliably estimated.

Below we present the data from the RXTE, confirming the large absorbing column, but also revealing large amplitude hard X-ray variability on a time scale of days. A distant absorber with an appreciable optical depth, subtending a large solid angle as seen by the nucleus, would smear out the rapid variability on time scales shorter than the light travel time through such an absorber. Given our data, we conclude that the optically thick absorber cannot be both distant and geometrically thick.

2. Observations: Spectrum and Variability

NGC 4945 was observed by the Rossi X-ray Timing Explorer (RXTE) satellite for about a month, starting on October 8, 1997. The observations included 38 pointings of ~ 2000 s each,

taken about once per day. The Proportional Counter Array (PCA) and High Energy X-ray Timing Experiment (HEXTE) data were reduced using standard procedures. For the PCA data, this included an extraction in the `standard2` mode using the `ftool saextrct`; the estimation of the background was done via `ftool pcabackest`, using the background model “L7” (model files `pca_bkgd_faint240_e03v03.mdl` and `pca_bkgd_faint17_e03v03.mdl`). For consistency, we use data only from 3 PCA detectors, which were active during all pointings. For HEXTE, the background is collected simultaneously by switching two halves of the array on- and off-source every 16 seconds; both source and background files were extracted using the `ftool seextrct`, and dead-time corrected using `ftool hxtdeadpha`. The total “good data” intervals were: 69,280 s for PCA, 18,364 s and 19,060 s for HEXTE clusters A and B.

The summed background-subtracted PCA and HEXTE files were fitted with a phenomenological model including a hard power law with low-energy photoelectric cutoff (using the cross-sections and abundances as given in Morrison & McCammon 1983) and a high-energy exponential cutoff (assumed to be at an energy E_c of 100 keV, in agreement with the high energy Seyfert spectra; e.g., Zdziarski et al. 1995). Our model also includes a Gaussian Fe K emission line, plus a soft component, which we modeled as another power law. In our fits, we used the PCA data corresponding to the energy range of 3 to 30 keV, and HEXTE data for 20 to 100 keV. The resulting fit (cf. Fig. 1) was essentially consistent with the Ginga / OSSE results of Done et al. (1996). The hard power law (with E_c of 100 keV) showed an energy index of 0.45 ± 0.1 , with absorption of $4.5 \pm 0.4 \times 10^{24} \text{ cm}^{-2}$, and an observed 10 - 50 keV flux of $1 \times 10^{-10} \text{ erg cm}^{-2} \text{ s}^{-1}$, with resulting $\chi^2 = 80.6/75$ d.o.f. The Fe K line energy was at $6.38 \pm 0.05 \text{ keV}$, with an intrinsic width σ of $0.37 \pm 0.13 \text{ keV}$, and a flux of $0.9 \times 10^{-4} \text{ photons cm}^{-2} \text{ s}^{-1}$. Allowing the cutoff to be unconstrained yielded the best fit of $90_{-30}^{+130} \text{ keV}$. Regarding the soft component, its energy power law index of was 0.57 ± 0.15 , with the 1 keV monochromatic flux of $0.001 \text{ photons cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1}$. It is important to note that the PCA field of view is about 1 deg^2 , so at least a fraction of the Fe K line and soft component flux could have arisen from the more extended (non-nuclear) region, a likely possibility given the starburst nature of the galaxy. Furthermore, given its modest flux, which over the 2 - 10 keV band is $6 \times 10^{-12} \text{ erg cm}^{-2} \text{ s}^{-1}$ - only three times that of the 1σ fluctuations of the Cosmic X-ray Background on the angular scale of the PCA field of view - we caution that any detailed spectral analysis of the PCA data for this soft component is unreliable. Nonetheless, we can clearly reject the hypothesis that the entire flux of this soft component is due to some kind of a “leaky absorber,” as it does not appear to vary.

The spectral analysis above confirmed the results of Done et al. (1996) that the source spectrum consists of the soft, relatively faint, unabsorbed component, a bright, heavily absorbed (hard) component, and a strong Fe K line. With that, we studied the variability of each component separately. The flux of the soft continuum component (below 8 keV) is consistent with being constant; this is also true of the Fe K line. The hard component (8 - 30 keV), on the other hand, is highly variable, with a factor of 4 change in 10 days, and a factor of 1.7 - 2 in 1 day between the minimum and maximum flux. This is plotted in Fig. 2. This light curve (collected with 3

PCUs, and binned on 1 day intervals) shows RMS variance (1σ) of 0.82 cts s^{-1} . Unfortunately, the source was too faint to study the variability with HEXTE.

We investigated if the variability could be due to instrumental effects, and specifically, imprecise background subtraction. To assess this, we also analyzed in an analogous manner the hard (8 – 30 keV) light curves binned in 1 day intervals of a cluster of galaxies Abell 754 (cf. Valinia et al. 1999) and a faint quasar PG1211+143, which shows very little flux in the PCA data above 8 keV (Netzer, Madejski, & Kaspi in prep.). Analysis of 9 data points collected over 9 days for A754 (from which no source variability is expected) yields $\sigma = 0.018 \text{ cts s}^{-1}$. For PG1211+143, we had 32 pointings spread nearly uniformly over 6 months. These data, where some intrinsic source variability may be present, yield $\sigma = 0.13 \text{ cts s}^{-1}$; we consider this a conservative upper limit to the instrumental effects, and thus deem the rapid hard X-ray variability of NGC 4945 with σ of 0.82 cts s^{-1} highly significant.

Could this variability be due to varying absorption? We examined this possibility by modeling separate spectra from high and low count rate observations. To improve statistics, we co-added the PCA spectra from a number of individual observations with highest and lowest count rates. The two resulting spectra were then modeled using the Monte Carlo absorption model discussed below. We assumed first that the intrinsic source spectrum (and normalization) is the same for both, but the absorption is different and, secondly, that the normalization of the intrinsic source has changed while the absorption stayed constant. The first hypothesis yielded $\chi^2 = 401/110 \text{ dof}$, while the second one yielded $\chi^2 = 143/110 \text{ d.o.f}$. This clearly shows that the variability is intrinsic to the unabsorbed nucleus.

3. Discussion

The variability we see in NGC 4945 is then entirely compatible with that expected from the *intrinsic* source, with no significant scattered delay by a distant material. If this absorber has an appreciable Compton thickness (as is the case here), and if it subtends a large solid angle to the X-ray source, then it should intercept and scatter a large fraction of the flux. If it is also distant from the nucleus, the light travel time effects will “wash out” any intrinsic variability on time scales shorter than the light travel time through the absorber. Conversely, a structure with much smaller scale height subtends a much smaller solid angle, making scattering less important. This also precludes the observed X-rays to be purely due to Compton reflection, as this would require a contrived geometry of the reflector with respect to the primary X-ray source: the reflector would have to be located very close to a completely covered central source. We note that by comparison, a well studied unabsorbed Seyfert 1 NGC 3516 showed X-ray variability with a similar fractional amplitude on $\sim 10\times$ longer time scales (Nandra & Edelson 1999) than seen in NGC 4945. While the black hole mass in NGC 3516 is not as well known, it is estimated to be $\sim 10^7 M_\odot$. The fact that the ratio of variability time scales is roughly the same as the ratio of nuclear masses – as expected for accreting black holes – further supports our conclusion that the hard X-ray

variability we see in NGC 4945 is intrinsic.

With the optical depth to electron scattering of a few, the shape of the absorption cutoff would be different than expected from pure photoelectric absorption, and the detailed shape of the emergent spectrum depends on the geometry. We model this numerically with a Monte Carlo code as given in Krolik, Madau, & Życki (1994), assuming a torus with square cross-section where the half-angle subtended by the torus, θ_0 , its optical depth to electron scattering τ_e , and the power law index of the incident energy spectrum α are free parameters (the Comptonization cutoff is set to 100 keV as above). The results of our fits (using the PCA data over the range of 3 – 20 keV, and HEXTE data as above) are shown in Table 1, where the 90% confidence regions on Γ and τ_e are typically 0.1.

A small scale height absorber ($\theta_0 \sim 10^\circ$) gives $\tau_e = 2.4$, compared to a large scale height ($\theta_0 \sim 80^\circ$) which gives $\tau_e = 2.1$. With an iron abundance of twice Solar, these fits change to $\tau_e = 1.7$, and $\tau_e = 1.5$, respectively. (Since the photoelectric cutoff present in our data is mainly sensitive to the column density of Fe, larger-than-Solar abundance of Fe would make us overestimate the true absorbing column if we assumed Solar abundances, and vice-versa.) While statistically these might marginally favor the large scale height absorber, we consider that all the fits are probably equally likely given that modeling the spectrum with a fixed cutoff energy may introduce systematic uncertainties. Our calculations include the Fe K emission line produced by the torus but we also include an additional Fe line (such as may be expected to arise in the photoionized scattering medium). Those calculations also imply that large ($> 4 \times$ Solar) Fe abundances can be excluded: they would imply a stronger Fe K line than is seen in our data. In reality, this limit is probably more stringent, given the fact that at least a fraction of the Fe K line originates in a more extended region.

These Monte Carlo results also give the distribution of the number of scatterings which the photons undergo before reaching the observer positioned in the equatorial plane. This is key in determining the solid angle subtended by the optically thick absorber, and thus its vertical size scale. Fig. 3 shows the fraction of the observed photons that underwent 0, 1, 2, 3, etc. scatterings before reaching the observer for eight values of θ_0 as discussed above (cf. Table 1), with the solid and dotted lines showing the results for Solar and twice Solar abundance of iron, respectively. The fraction of photons which arrive without being scattered is 19% and 63% respectively for a “thick” ($\theta_0 = 80^\circ$) and “skinny” ($\theta_0 = 10^\circ$) torus. For $2 \times$ Solar abundance of Fe these numbers are 32% and 75%. The data in Fig. 2 imply that fewer than 40% of the observed photons are scattered over path lengths longer than 1 light day, so the half-angle subtended by the optically thick material is less than $\sim 10^\circ$. This implies a rather small scale height, and perhaps is due to the same material which produces the H₂O maser emission (Greenhill et al. 1997).

The details of the geometry of Seyfert 2s are important in the assessment of their contribution to the Cosmic X-ray background, as the heavily absorbed AGN were postulated to make up the bulk of it (cf. Krolik et al. 1994; Madau, Ghisellini, & Fabian 1994; Comastri et al. 1995). The

value of θ_0 of 10° is marginally consistent with the torus geometry inferred from recent observations at hard X-ray energies. These observations show that Seyfert 2s outnumber Seyfert 1s by a factor of 4:1, while more than a quarter of Seyfert 2s have a column which is optically thick (see Giommi et al. 1998, and Gilli, Risaliti, & Salvati 1999 and references therein). Assuming a single geometry for the Seyfert nuclei where the torus has a rectangular cross-section, we can attempt to reproduce these ratios by assuming that if the central flux barely “grazes” the torus, we classify the object as any Seyfert 2, while an object is an optically thick Seyfert 2 only if the line of sight encounters the entire radial distance in the torus. In this context, requiring such 1:4:1 ratio would then imply that the torus is somewhat flattened, with outer radius of $6.5 \times$ its inner radius and equatorial height of $1.3 \times$ its inner radius. In this scenario, all Seyfert 2s are then seen at angles smaller than $\sim 50^\circ$ from the plane, while the optically thick Seyfert 2s are confined to angles of $\leq 12^\circ$. We repeated the Monte-Carlo calculations with this rectangular geometry and find that the fraction of scattered photons is $\sim 50\%$, still too large to match the observed hard X-ray variability. A further problem arises if this is indeed a universal geometry for all Seyferts as the Thomson depth of the absorber is large, ~ 1.7 in the more restrictive $2 \times$ Solar case. This absorber reduces the unabsorbed flux \sim five-fold or more, and this is not consistent with the 1:4:1 ratio, by a large margin. We thus conclude that a population of AGN with geometry very nearly that of NGC 4945 cannot make up the CXB. Instead, significantly larger fraction of the heavily obscured AGN is required (implying a large solid angle subtended by the absorber) than implied from the rather small θ_0 inferred by us. One plausible scenario would have the local optically thick Seyfert 2s surrounded by absorbers that already collapsed to an accretion disk, while in the more distant objects – in the earlier stages of evolution – such absorbers had larger vertical extent.

Alternately, the absorption could come from a structure which is much closer to the nucleus. The variability limits impose constraints on the amount of scattered flux that is lagged on time scales of more than 1 day, and thus they do not constrain the height of any structure which is $\ll 1$ light day from the X-ray source (corresponding to a distance of $\ll 10^4$ Schwarzschild radii for the mass of the black hole of $\sim 10^6 M_\odot$). While a very geometrically thick accretion disk cannot be ruled out from our data, there are considerable theoretical difficulties in maintaining a structure with a large height scale. It is far easier to envisage a structure with a small height scale such an accretion disk with outer regions somewhat thickened by instabilities resulting from radiation pressure warping (cf. Maloney, Begelman, & Pringle 1996).

With these arguments for the Thomson-thick absorber subtending a small solid angle in NGC 4945, we can now estimate the true luminosity of the source. The Monte Carlo simulations show that the intrinsic flux must be substantially larger than that observed (by a factor of ϵ^{τ_e} , or $\times 11$ for Solar abundances) because any scattered nuclear photons are lost from the line of sight. The 1-500 keV flux, corrected for photoelectric absorption alone, is $\sim 5 \times 10^{-10}$ erg cm $^{-2}$ s $^{-1}$, so correcting for the Thomson opacity yields a 1-500 keV intrinsic X-ray luminosity of 10^{43} erg s $^{-1}$. Assuming that the thermal (opt/UV/EUV) emission from the accretion disk is roughly equal to the hard X-ray emission gives a total bolometric luminosity of the nucleus of $\sim 2 \times 10^{43}$

erg s^{-1} . With this, and M_{BH} of $1.4 \times 10^6 M_{\odot}$, the source is radiating at $\sim 10\%$ of the Eddington luminosity; even if the abundances are twice-Solar (which yields τ_e of 1.7), L/L_{Edd} is at least $\sim 5\%$. (We note that similar L/L_{Edd} was also inferred by Greenhill et al. (1997), but the discovery of the rapid hard X-ray variability allows us to determine the source luminosity more accurately, as now we know that relatively few photons are scattered *back* to the line of sight.) NGC 4945 is one of the few AGN where this quantity can be calculated robustly, since the mass of the central object is *known*, although we are aware that the value of its central mass is not as accurate as for the famous megamaser NGC 4258; the resulting uncertainty in the estimate of L/L_{Edd} may be a factor of 2, comparable to the effects of the unknown Fe abundance or the ratio of $L_{\text{Tot}}/L_{\text{X-ray}}$. The resulting L/L_{Edd} is comparable to that inferred for the well studied Seyfert 2 NGC 1068, although since the absorber is completely opaque even to hard X-rays, the central luminosity can be estimated only indirectly. These two AGN are at the opposite end of the scale to NGC 4258, which radiates at $\sim 10^{-4} L_{\text{Edd}}$ or less (Lasota et al. 1996). The mass accretion rates inferred from those values of L/L_{Edd} put strong constraints on possible underlying radiation mechanisms. While the recently popular advective disk models can produce X-ray hot flows up to about 10% of the Eddington limit, these collapse at higher L/L_{Edd} : our data show that NGC 4945 lies perilously close to this limits.

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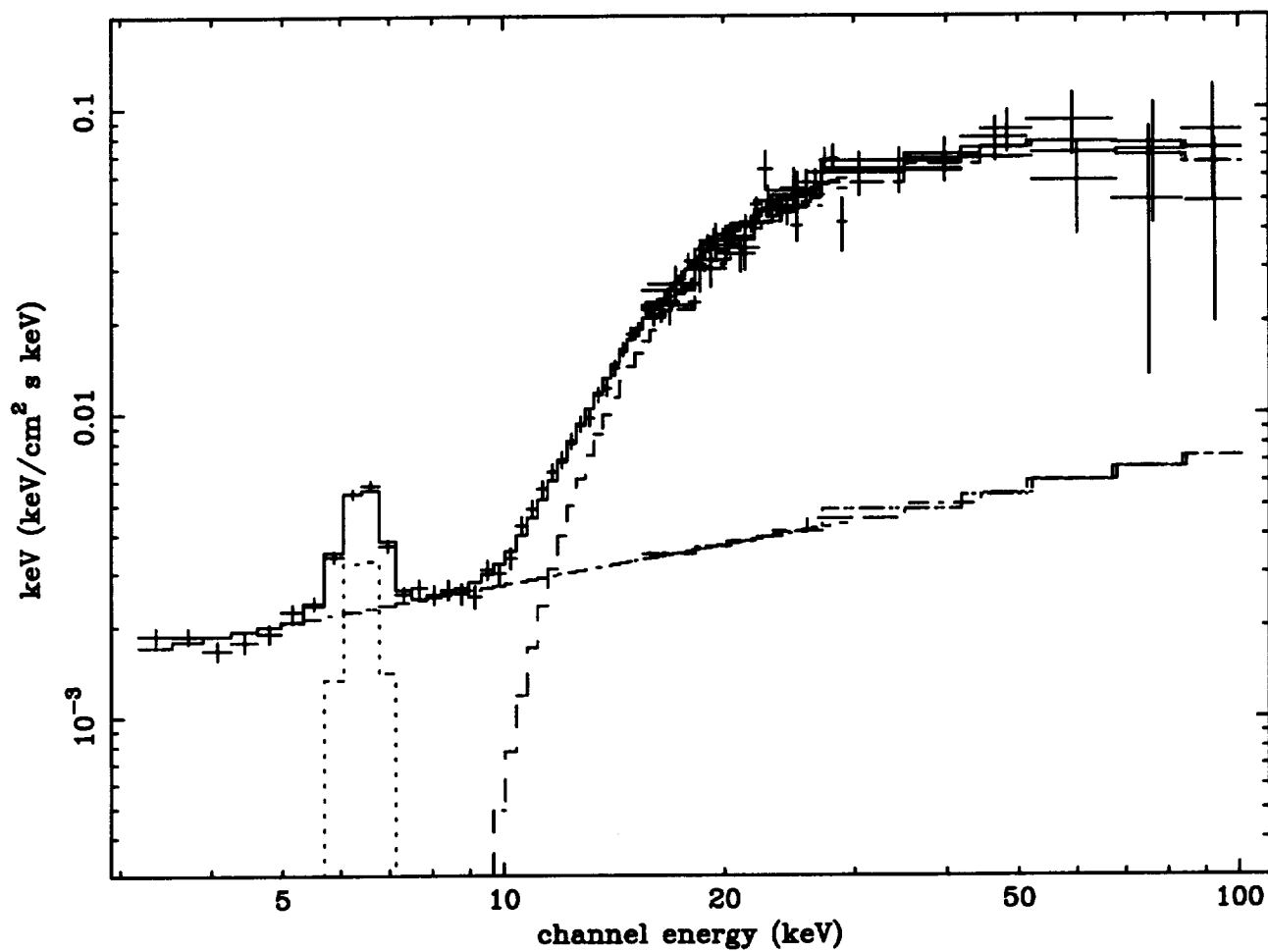
Figure Captions

Fig. 1: Broad-band unfolded X-ray spectrum of NGC 4945 as measured with the RXTE PCA and HEXTE instruments. The data were fitted with a phenomenological model which includes a hard power law component photo-electrically absorbed by neutral gas with Solar abundances at a column of $4.5 \pm 0.3 \times 10^{24} \text{ cm}^{-2}$, with photon spectral index $\Gamma = 1.45^{+0.1}_{-0.1}$, exponentially cutting off at 100 keV, plus a non-variable soft component (assumed to be a power law), and a Fe K line. The observed 8 – 30 keV flux of the hard component is $5 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$.

Fig. 2: Hard X-ray light curve of the Seyfert 2 galaxy NGC 4945 measured with the RXTE PCA instrument, showing a rapid, large amplitude flux variability. Plotted are data from all three layers of three PCA detectors that were turned on during all pointings, over the energy channels nominally corresponding to the range of 8 – 30 keV.

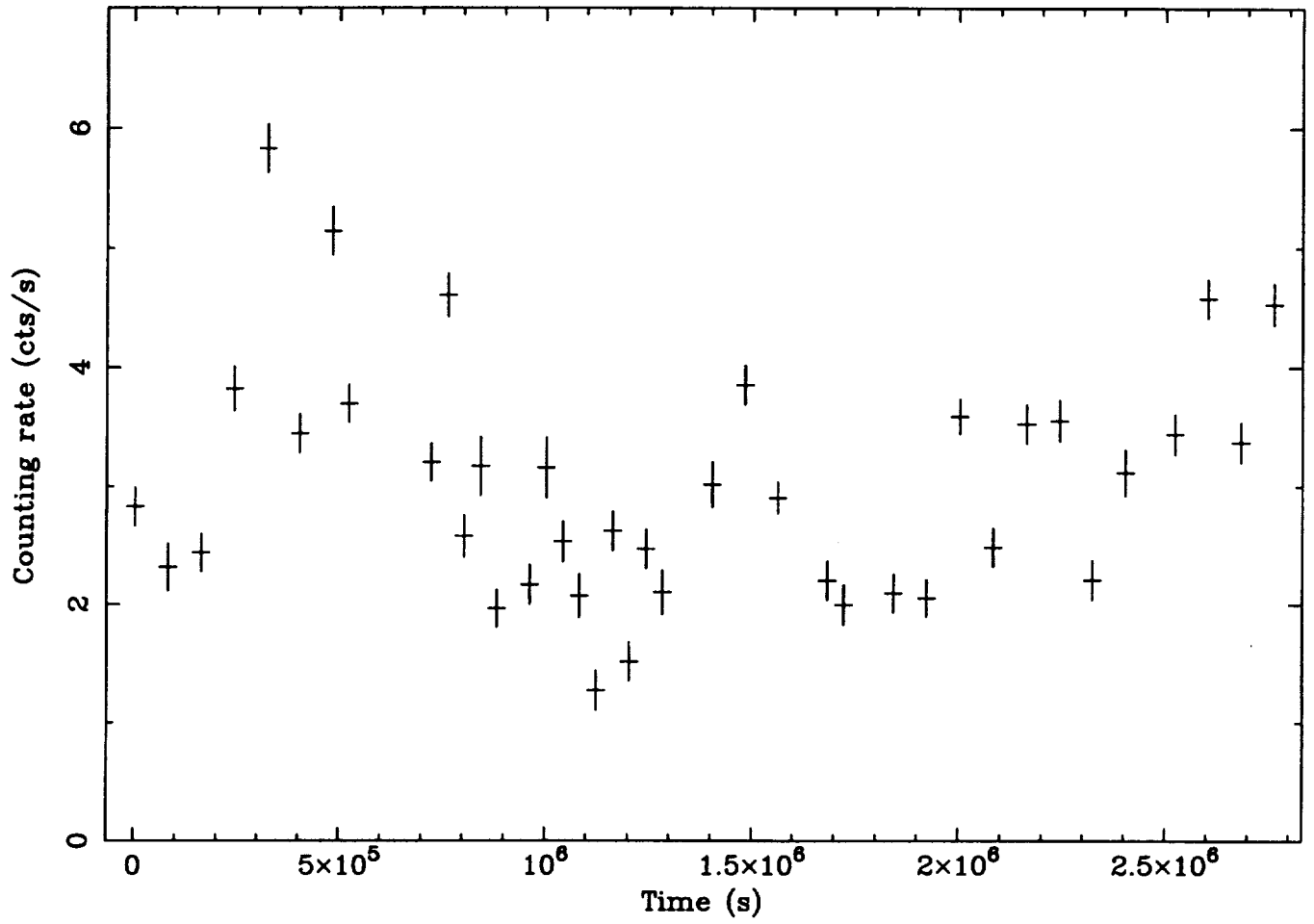
Fig. 3: Fraction of the observed photons reaching an observer located in the equatorial plane of a torus plotted against the number of scatterings that those photons encountered before reaching an observer. The angle given in each panel is the vertical half-angle θ_0 subtended by the torus as seen from the central source. Iron abundance (relative to Solar) is assumed to be 1 (solid line) and 2 (dotted line), with τ_e equal to the best fit value for a given Fe abundance and θ_0 (cf. Table 1). Since the fractional amplitude of variability on short time scales (cf. Fig. 2) is large ($> 60\%$), θ_0 of the optically thick structure must be small, so that majority of the photons reaching an observer are not scattered.

RXTE PCA and HEXTE unfolded spectrum of the Seyfert 2 galaxy NGC 4945



NGC_4945

Bin time: 0.4000E+05 s



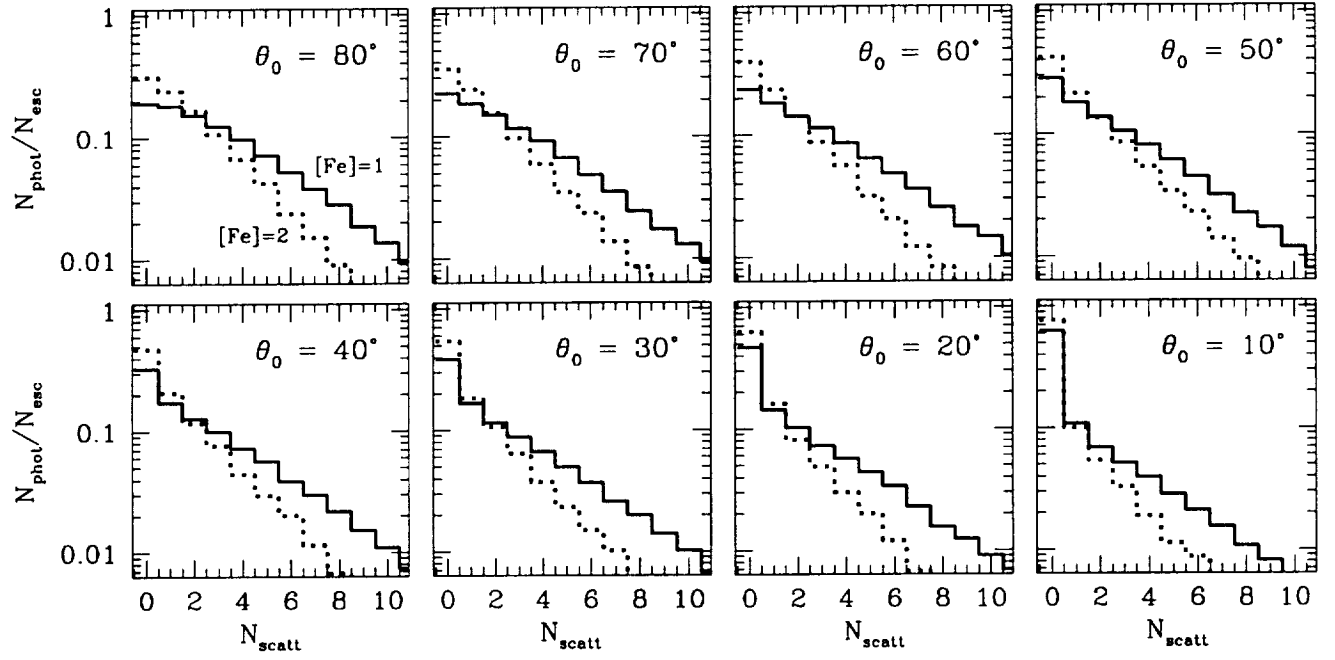


Table 1. Results of Monte Carlo Fits to the RXTE Data for NGC 4945 for Assumed Solar and $2\times$ Solar Fe Abundances

Assumed torus half-angle θ_0 (degrees)	Fitted spectral index α		Fitted optical depth τ_e		χ^2 76 d.o.f.		Fraction of detected <i>unscattered</i> photons	
	$A_{\text{Fe}} = 1$	$A_{\text{Fe}} = 2$	$A_{\text{Fe}} = 1$	$A_{\text{Fe}} = 2$	$A_{\text{Fe}} = 1$	$A_{\text{Fe}} = 2$	$A_{\text{Fe}} = 1$	$A_{\text{Fe}} = 2$
80	0.7	0.75	2.1	1.5	68.5	68.5	19%	31%
70	0.7	0.75	2.1	1.5	69.8	68.3	22%	35%
60	0.7	0.75	2.2	1.5	71.5	72.9	24%	39%
50	0.8	0.75	2.2	1.6	70.9	69.6	28%	42%
40	0.7	0.75	2.2	1.6	70.9	72.7	33%	48%
30	0.8	0.75	2.3	1.6	76.6	74.6	38%	55%
20	0.8	0.75	2.4	1.6	74.5	77.0	47%	63%
10	0.8	0.8	2.4	1.7	75.4	79.4	63%	76%